DESIGN AND ANALYSIS OF SOME CONFOUNDED QUALITATIVE-*CUM*-QUANTITATIVE EXPERIMENTS

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Introduction

Experiments involving factors with both quantitative and qualitative levels are known to have distinct advantages over a series of simple experiments each designed to test separately qualities at different levels. As early as in 1927 (Eden and Fisher, 1929) a qualitative-*cum*-quantitative experiment was laid out at the Rothamsted Experimental Station in which different forms and levels of potassic fertilizers together with nitrogen at different levels were studied.

The first detailed account of such experiments was given by Fisher (1935). He points out that the assumption of additive effects of qualities and quantities (additive model) is not wholly satisfactory and proposes instead the model that quality differences may be regarded proportional to quantity applied (proportional model).

Williams (1952) discusses a much more general problem where the joint effect of two or more factors are not additive. He proposes a model in which the effect of one factor (quality) is considered to be proportional at different levels of the other factor (quantity). The proportions are, however, not simply those of the quantities applied as proposed by Fisher.

The investigation of the choice of model for the analysis of such experiments has been carried out further by Kempthorne (1951) who has discussed how the knowledge of response curves helps in the choice of the appropriate model.

Cox and Cochran (1950) have also discussed the problem of the subdivision of the sum of squares of an interaction table on the hypothesis of proportionality.

There are other types of qualitative-*cum*-quantitative experiments which involve only non-zero levels. Rayner (1953) on the basis of the method proposed by Fisher (1951) has discussed the method of analysis for such types of experiments involving no confounding. His method of analysis is based on the technique of fitting constants.

The system of confounding of symmetrical and asymmetrical qualitative-*cum*-quantitative experiments involving dummies often present certain novel features not ordinarily met with in designs involving quantitative levels of different factors. Mathematical details of analysis of such experiments are lacking in the current literature. The analysis can best be made by fitting constants as suggested by Yates (1933). The main object of this paper is to explain the analysis of such experiments by discussing possible types of confounding and presenting the analysis both under the additive and proportional models for the following types of commonly used experiments involving dummy treatments:

'n'		'p'
Quantities	Qualities	Quantities or Qualities
3	3	3
3	2	2
3	3	2
3	2	3
3	4	2

1·3[3]. Qualitative-*cum*-Quantitative Experiments

Let the three factors be:

(i) 3 quantities of 'n'—n_0, n_1 and n_2 in the ratio 0 : 1 : 2.

(ii) 3 qualities of 'n'—q_0, q_1, q_2.

(iii) 3 quantities or qualities of 'p'—p_0, p_1 and p_2.

Ordinarily a 3^3 design is arranged in 9 plot blocks confounding a component of second order interaction. Several types of confounding are possible. As pointed out by Yates (1933), some of the possible types of confounding are derivable from the classical system of confounding for ordinary factorial designs by using dummy treatments where necessary. There are other types not so derivable but equally efficient. A method of obtaining these designs is discussed below.

1.1. *Systems of confounding—9 plot blocks*

If the treatments be arranged in blocks of 9 plots and N, P and NP are to be kept free from block differences, each block must contain every possible combination of three quantities or qualities of 'p' and three levels of 'n'. In fact, if in addition to N, P and NP, Q and NQ are also be kept free from block differences, then at each level of 'n' one plot must receive 'n' through q_0, another through q_1, and the third through q_2. The only way in which blocks can differ consists in the manner in which three qualities of 'n' are assigned to plots receiving 0, 1, 2 quantities of 'n'. The question of application of 'n' through different qualities does not arise at zero quantity since the three combination of zero quantity of 'n' with 0, 1, 2 quantities or qualities of 'p' are p_0, p_1 and p_2. Considering plots receiving n_1 dose of 'n', each block contains treatments n_1p_0, n_1p_1, n_1p_2. In one of these combinations 'n' is to be supplied through q_0, in another through q_1, and in the third through q_2. q_0, q_1 and q_2 can be allotted to these three different treatments in six different ways which are divisible into two cyclic orders as shown below:

$$\left.\begin{matrix} n_1q_0p_0 & n_1q_1p_0 & n_1q_2p_0 \\ n_1q_1p_1 & n_1q_2p_1 & n_1q_0p_1 \\ n_1q_2p_2 & n_1q_0p_2 & n_1q_1p_2 \end{matrix}\right\} \qquad \left\{\begin{matrix} n_1q_0p_0 & n_1q_1p_0 & n_1q_2p_0 \\ n_1q_2p_1 & n_1q_0p_1 & n_1q_1p_1 \\ n_1q_1p_2 & n_1q_2p_2 & n_1q_0p_2 \end{matrix}\right.$$

Thus if one of the blocks contains the treatments $n_1q_0p_0$, $n_1q_1p_1$, $n_1q_2p_2$ then the second block will contain treatments $n_1q_1p_0$, $n_1q_2p_1$, $n_1q_0p_2$ and the third block will contain the treatments $n_1q_2p_0$, $n_1q_0p_1$, $n_1q_1p_2$ } Set I

or alternatively if one of the blocks contains the treatments $n_1q_0p_0$, $n_1q_2p_1$, $n_1q_1p_2$, then the second block will contain the treatments $n_1q_1p_0$, $n_1q_0p_1$, $n_1q_2p_2$, and the third block will contain the treatments $n_1q_2p_0$, $n_1q_1p_1$, $n_1q_0p_2$. } Set II

Similarly for plots receiving n_2 dose of 'n'. If one of the blocks contains the treatments $n_2q_0p_0$, $n_2q_1p_1$, $n_2q_2p_2$ then the second block will contain the treatments $n_2q_1p_0$, $n_2q_2p_1$, $n_2q_0p_2$, and the third block will contain the treatments $n_2q_2p_0$, $n_2q_0p_1$, $n_2q_1p_2$ } Set III

or alternatively if one of the blocks contains the treatments $n_2q_0p_0$, $n_2q_2p_1$, $n_2q_1p_2$ then the second block will contain the treatments $n_2q_1p_0$, $n_2q_0p_1$, $n_2q_2p_2$ and the third block will contain the treatments $n_2q_2p_0$, $n_2q_1p_1$, $n_2q_0p_2$. } Set IV

The possible designs are obtained by combining set I with sets III and IV and set II with sets III and IV. Set I can be combined with set III in 6 different ways. Similarly set I can be combined with set IV in 6 different ways. Thus combination of set I with sets III and IV gives 12 designs. Similarly 12 more designs for set II and in all 24 different designs which are given in Appendix I.

Out of these 24 different designs, design Nos. 2, 3, 8 and 9 are the designs which are derivable from the classical system of confounding by using dummy treatments where necessary.

1.2. *Confounded effects*

It is noticed that in case of all these 24 different designs, both QP and NQP are affected by block differences. It is only these 8 degrees of freedom which complicate the analysis. In order to see what has happened to this group of comparison, Fisher (1935) suggested that these 8 d.f. should be split up into two components, *viz.*, one with 4 d.f. obtained from the interaction QP at n_1 dose of 'n' and the other also with 4 d.f. from the same interaction at n_2 dose of 'n'. These four degrees of freedom each at n_1 and n_2 doses can further be split up into usual orthogonal components I and J for n_1 dose and I' and J' for n_2 dose. The components affected in different designs obtained earlier have been shown in the table on next page.

It is important to note that the degrees of freedom affected by block differences in all the designs in the same group are same though they differ in respect of block contents. Also it can be easily seen that group 1st $\equiv$ group 5th; group 2nd is $\equiv$ group 6th; group 3rd is $\equiv$ group 7th; and group 4th is $\equiv$ group 8th.

Group No.	Designs included	Degrees of freedom affected
1st	1, 2 and 3	I component of QP at n_1 (2 d.f.) I' component of QP at n_2 (2 d.f.)
2nd	4, 5 and 6	I component of QP at n_1 (2 d.f.) J' component of QP at n_2 (2 d.f.)
3rd	7, 8 and 9	J component of QP at n_1 (2 d.f.) J' component of QP at n_2 (2 d.f.)
4th	10, 11 and 12	J component of QP at n_1 (2 d.f.) I' component of QP at n_2 (2 d.f.)
5th	13, 14 and 15	I component of QP at n_1 (2 d.f.) I' component of QP at n_2 (2 d.f.)
6th	16, 17 and 18	I component of QP at n_1 (2 d.f.) J' component of QP at n_2 (2 d.f.)
7th	19, 20 and 21	J component of QP at n_1 (2 d.f.) J' component of QP at n_2 (2 d.f.)
8th	22, 23 and 24	J component of QP at n_1 (2 d.f.) I' component of QP at n_2 (2 d.f.)

1·3. *Analysis of 3^3 experiments in 9 plot blocks—Additive model. Single replicate design*:

Under the additive assumption the model to be used is:

$$y_{ijk} = \mu + N_i + \delta Q_j + \delta (NQ)_{ij} + P_k + NP_{i+k} + NP^2_{i+2k} + \delta' (I_{j+2k} + J_{j+k}) + \delta'' (I'_{j+2k} + J'_{j+k}) + \beta_l + e_{ijkl},$$

where the notations NP, NP^2, etc., are the same as used by Kempthorne (1951) excepting that I_{j+2k}, etc., have the meaning defined earlier and y_{ijk} is the yield from the treatment combination $n_i q_j p_k$ and β_l the effect of block containing treatment combination $n_i q_j p_k$.

The restriction on the parameters are:

$$\begin{aligned} \delta &= 0 \quad \text{for} \quad i = 0, \quad \text{otherwise} \quad \delta = 1 \\ \delta' &= 1 \quad \text{for} \quad i = 1, \quad \text{otherwise} \quad \delta' = 0 \\ \delta'' &= 1 \quad \text{for} \quad i = 2, \quad \text{otherwise} \quad \delta'' = 0 \end{aligned}$$

and

$$\left.\begin{array}{llll}\sum_{i=0}^{2} N_i=0 & \sum_{j=0}^{2} (NQ)_{ij}=0 & NP_0^2+NP_1^2 +NP_2^2=0 & I'_0+I'_1+I'_2=0 \\ \sum_{j=0}^{2} Q_j=0 & \sum_{k=0}^{2} P_k=0 & I_0+I_1+I_2=0 & J_0'+J_1'+J_2'=0 \\ \sum_{i=1,2} (NQ)_{ij}=0 & NP_0+NP_1 +NP_2=0 & J_0+J_1+J_2=0 & \sum_{l=1}^{3} \beta_l=0\end{array}\right\} \quad (1)$$

e_{ijkl}'s are independently normally distributed with mean zero and same variance. Utilising the above-mentioned restrictions the model simplifies to:

$$\begin{aligned} y_{ijk} = \mu + N_i + \delta Q_j + \delta (-1)^i (NQ)_j + P_k + NP_{i+k} \\ + NP^2{}_{i+2k} + \delta' (I_{j+2k} + J_{j+k}) + \delta'' (I'_{j+2k} + J'_{j+k}) \\ + \beta_l + e_{ijkl}. \end{aligned}$$

The restrictions on the parameters remain the same excepting that

$$\sum_{i=1,\,2} (NQ)_{ij} = 0 \quad \text{and} \quad \sum_{j=0}^{2} (NQ)_{ij} = 0$$

are to be replaced by

$$\sum_{j=0}^{2} (NQ)_j = 0.$$

Taking any one of the 24 designs, say No. 7, where the components J of QP at n_1 and J' of QP at n_2 are affected by block differences, the normal equations for estimating the components affected by block differences come out as:

$$\left.\begin{array}{l} 3\mu + 3N_1 + 3J_0 + 3\beta_1 = y_{100} + y_{121} + y_{112} \\ 3\mu + 3N_1 + 3J_1 + 3\beta_2 = y_{110} + y_{101} + y_{122} \\ 3\mu + 3N_1 + 3J_2 + 3\beta_3 = y_{120} + y_{111} + y_{102} \end{array}\right\} \quad (2)$$

$$\left.\begin{array}{l} 3\mu + 3N_2 + 3J_0' + 3\beta_1 = y_{200} + y_{221} + y_{212} \\ 3\mu + 3N_2 + 3J_1' + 3\beta_2 = y_{210} + y_{201} + y_{222} \\ 3\mu + 3N_2 + 3J_2' + 3\beta_3 = y_{220} + y_{211} + y_{202} \end{array}\right\} \quad (3)$$

Normal equations for block parameters are:

$$\left.\begin{aligned} 9\mu + 3J_0 + 3J_0' + 9\beta_1 &= B_1 \\ 9\mu + 3J_1 + 3J_1' + 9\beta_2 &= B_2 \\ 9\mu + 3J_2 + 3J_2' + 9\beta_3 &= B_3 \end{aligned}\right\} \quad (4)$$

where B_l is the total of l-th block.

For any other design the normal equations remain the same excepting that the J and J' components in these equations get replaced by those which are affected.

It is important to point out that the estimates of J (2 d.f.) and J' (2 d.f.) obtained by solving sets (2) and (3) with the help of (4) will not be orthogonal. To overcome this difficulty some joint estimates of J and J' which are not only mutually orthogonal but orthogonal to all other effects are obtained as follows:

From set (2) and set (3) we have:

$$\left.\begin{aligned} \widehat{J_0 - J_0'} &= \frac{(y_{100} + y_{121} + y_{112}) - (y_{200} + y_{221} + y_{212})}{3} \\ &\quad - \frac{1}{9}(y_{1..} - y_{2..}) \\ \widehat{J_1 - J_1'} &= \frac{(y_{110} + y_{101} + y_{122}) - (y_{210} + y_{201} + y_{222})}{3} \\ &\quad - \frac{1}{9}(y_{1..} - y_{2..}) \\ \widehat{J_2 - J_2'} &= \frac{(y_{120} + y_{111} + y_{102}) - (y_{220} + y_{211} + y_{202})}{3} \\ &\quad - \frac{1}{9}(y_{1..} - y_{2..}) \end{aligned}\right\}. \quad (5)$$

Estimate given by (5) is a joint estimate accounting for 2 d.f. out of 4 d.f. from both J and J' components. As we can not be sure whether this joint estimate is precisely a component of QP or NQP (defined in the usual way), so the estimate given by (5) is $\left.{}^{QP}_{NQP}\right\}$ 2 unconfounded degrees of freedom.

Adding the corresponding equations in set (2) and set (3) and solving the resultant set of equations with the help of set (4) we have;

$$\left.\begin{aligned}
\widehat{J_0+J_0'} &= \frac{y_{100}+y_{121}+y_{112}+y_{200}+y_{221}+y_{212}}{3} \\
&\quad -\frac{2}{3}(n_0 \text{ plots in } B_1)-\frac{1}{9}(y_{1..}+y_{2..}-2y_{0..}) \\
\widehat{J_1+J_1'} &= \frac{y_{110}+y_{101}+y_{122}+y_{210}+y_{201}+y_{222}}{3} \\
&\quad -\frac{2}{3}(n_0 \text{ plots in } B_2)-\frac{1}{9}(y_{1..}+y_{2..}-2y_{0..}) \\
\widehat{J_2+J_2'} &= \frac{y_{120}+y_{111}+y_{102}+y_{220}+y_{211}+y_{202}}{3} \\
&\quad -\frac{2}{3}(n_0 \text{ plots in } B_3)-\frac{1}{9}(y_{1..}+y_{2..}-2y_{0..})
\end{aligned}\right\} \quad (6)$$

Where 'n_0 plots in B_1' means yield of those plots in the 1st block which receive n_0 level of 'n'.

Estimate given by (6) is $\left.{}^{QP}_{NQP}\right\}$ 2 partially confounded degrees of freedom.

The sum of squares due to unconfounded effects can be obtained in the usual way taking care of the definition of I and I' components as used in the model. The sum of squares due to $\left.{}^{QP}_{NQP}\right\}$ 2 unconfounded degrees of freedom and $\left.{}^{QP}_{NQP}\right\}$ 2 partially confounded degrees of freedom are

$$\frac{3}{2}\sum_{i=0}^{2}(\widehat{J_i-J_i'})^2 \quad \text{and} \quad \frac{1}{2}\sum_{i=0}^{2}(\widehat{J_i+J_i'})^2$$

respectively.

The relative information of $\left.{}^{QP}_{NQP}\right\}$ 2 partially confounded degrees of freedom with respect to an unconfounded design is $\frac{1}{3}\sigma_{27}^2/\sigma_9^2$, where σ_{27}^2 and σ_9^2 are the variances per plot in case of designs with 27 plots and 9 plots per block respectively.

Two Replications.—Two replications can be chosen in the following alternative ways:

(i) Choose one of the 24 possible designs and repeat it in both the replications.

(ii) Choose two designs from the same group.

(iii) Choose one design from one group and the other from a group not confounding the same degrees of freedom, *e.g.*, design No. 1 and 7 and not 1 and 14, Analysis for choice No. (i) is simple as the same degrees of freedom are affected by block differences in both the replications. Analysis for choice No. (ii) is complicated because the same two degrees of freedom are being affected by block differences in two different ways. With choice No. (iii), information will be available as $\left.\begin{matrix}QP\\NQP\end{matrix}\right\}$ 4 unconfounded degrees of freedom and $\left.\begin{matrix}QP\\NQP\end{matrix}\right\}$ 4 partially confounded degrees of freedom. It can thus be concluded that the best choice for two replications is to choose any one of the 24 possible designs and repeat it in both the replications.

Three Replications.—As pointed out earlier, the 24 possible designs can be classified into 4 different groups. The best choice for three replications is to choose one of the four groups (1st to 4th).

Taking any one of the four groups, say 3rd, where the components J of QP at n_1 and J' of QP at n_2 are affected by block differences, the normal equations for estimating the components affected by block differences come out as:

$$\left.\begin{aligned}&9\mu + 9N_1 + 9J_0 + 3(\beta_{11} + \beta_{12} + \beta_{13})\\&\qquad = y_{100.} + y_{121.} + y_{112.}\\&9\mu + 9N_1 + 9J_1 + 3(\beta_{21} + \beta_{22} + \beta_{23})\\&\qquad = y_{110.} + y_{101.} + y_{122.}\\&9\mu + 9N_1 + 9J_2 + 3(\beta_{31} + \beta_{32} + \beta_{33})\\&\qquad = y_{120.} + y_{111.} + y_{102.}\end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned}&9\mu + 9N_2 + 9J_0' + 3(\beta_{11} + \beta_{32} + \beta_{23})\\&\qquad = y_{200.} + y_{221.} + y_{212.}\\&9\mu + 9N_2 + 9J_1' + 3(\beta_{21} + \beta_{12} + \beta_{33})\\&\qquad = y_{210.} + y_{201.} + y_{222.}\\&9\mu + 9N_2 + 9J_2' + 3(\beta_{31} + \beta_{22} + \beta_{13})\\&\qquad = y_{220.} + y_{211.} + y_{202.}\end{aligned}\right\} \tag{8}$$

where y_{ijkl} is the yield of the treatment combination $n_iq_jp_k$ in the l-th replication and dot replacing a suffix meaning summation over that suffix. β_{ml} is the effect of m-th block in l-th replication and γ_l the effect of l-th replication.

Normal equations for block parameters are:

$$\left.\begin{aligned}9\mu + 9\gamma_l + 9\beta_{1l} + 3J_0 + 3J_0' &= B_{1l}\\9\mu + 9\gamma_l + 9\beta_{2l} + 3J_1 + 3J_1' &= B_{2l}\\9\mu + 9\gamma_l + 9\beta_{3l} + 3J_2 + 3J_2' &= B_{3l}\end{aligned}\right\} \tag{9}$$

for $l = 1$, 2 and 3

Solving set (7) with the help of set (9), we have:

$$\left.\begin{aligned}\hat{J}_0 &= \frac{1}{18} \sum_{B_{11}, B_{12}, B_{13}} (2n_1 \text{ plots} - n_2 \text{ plots} - n_0 \text{ plots}) - \frac{27}{18} \hat{N}_1 \\ \hat{J}_1 &= \frac{1}{18} \sum_{B_{21}, B_{22}, B_{23}} (2n_1 \text{ plots} - n_2 \text{ plots} - n_0 \text{ plots}) - \frac{27}{18} \hat{N}_1 \\ \hat{J}_2 &= \frac{1}{18} \sum_{B_{31}, B_{32}, B_{33}} (2n_1 \text{ plots} - n_2 \text{ plots} - n_0 \text{ plots}) - \frac{27}{18} \hat{N}_1\end{aligned}\right\} .$$

Where 'n_i plots' stands for the yield from the plots receiving n_i dose of 'n' and $\sum_{B_{11}, B_{12}, B_{13}}$ stands for summation over those blocks totals of which have been shown under the summation sign.

Similarly set (8) gives:

$$\left.\begin{aligned}\hat{J}_0' &= \frac{1}{18} \sum_{B_{11}, B_{32}, B_{23}} (2n_2 \text{ plots} - n_1 \text{ plots} - n_0 \text{ plots}) - \frac{27}{18} \hat{N}_2 \\ \hat{J}_1' &= \frac{1}{18} \sum_{B_{21}, B_{12}, B_{33}} (2n_2 \text{ plots} - n_1 \text{ plots} - n_0 \text{ plots}) - \frac{27}{18} \hat{N}_2 \\ \hat{J}_2' &= \frac{1}{18} \sum_{B_{31}, B_{22}, B_{13}} (2n_2 \text{ plots} - n_1 \text{ plots} - n_0 \text{ plots}) - \frac{27}{18} N_2\end{aligned}\right\} .$$

The sum of squares due to unconfounded effects can be obtained in the usual way. The sum of squares due to partially confounded effects J of QP at n_1 and J' of QP at n_2 are $6 \sum_{i=0}^{2} \hat{J}_i^2$ and $6 \sum_{i=0}^{2} \hat{J}_i'^2$ respectively.

The relative information of both J of QP at n_1 and J' of QP at n_2 with respect to an unconfounded design is $\frac{2}{3}\, \sigma_{27}^2/\sigma_9^2$, where σ_{27}^2 and σ_9^2 are the variances per plot in case of designs with 27 plots and 9 plots per block respectively.

1.4. *Analysis of 3^3 experiments in 9 plot blocks—Proportional model*

The analysis under the proportional model differs from that under the additive model only in the sum of squares due to Q and NQ which is simple as these effects are not affected by block differences. In case the non-zero quantities of 'n' are in the ratio $1 : \alpha$, weights 1 and α

may be used instead of 1 and 2 in the calculation of the sum of squares due to Q and NQ.

1.5. *An important note*

In situations where a 3^3 qualitative-*cum*-quantitative experiment in one replication is to be repeated over years, it is better to carry out the investigation for a period of three or a multiple of three years. In case the investigation is over a period of three years, the best choice would be to take the designs belonging to the same group such that a different design out of the chosen group is used every year. If the investigation is to be carried out for a period of six years or nine years or more, then a different group should preferably be used for years 1 to 3; 4 to 6; 7 to 9, etc. This procedure has the advantage that the combined analysis will furnish information on QP at n_1 and QP at n_2 separately and not as a combined effect of QP and NQP as available with any other choice.

2. 3×2×2 QUALITATIVE-*cum*-QUANTITATIVE EXPERIMENTS

In qualitative-*cum*-quantitative experiments there should be at least two non-zero levels of the factor which is being tried in different forms. Thus the possible 3×2×2 qualitative-*cum*-quantitative experiments are of the type involving 3 quantities of 'n' — (n_0, n_1, n_2, in the ratio 0 : 1 : 2); 2 qualities of 'n' — (q_0, q_1) and 2 quantities or qualities of 'p' (p_0, p_1).

Confounded 3×2×2 qualitative-*cum*-quantitative designs in 6 plot blocks are derivable from the classical 3×2×2 in 6 plot block designs given by Yates (1937) by using dummy treatment where necessary. In case of qualitative-*cum*-quantitative experiments two out of these three replications for the usual 3×2×2 in 6 plot block designs become identical, thereby giving rise to only two different replications for the design and these have been presented in Appendix II. In what follows these two replications will be referred to as designs I and II respectively. These two designs could also be obtained otherwise by a method similar to the one used in section 1.1.

2.1. *Confounded effects*

At the first instance it might appear that with 6 plot blocks, both QP and NQP will be affected by block differences. But it can easily be seen that this is not so under the additive model. Under the additive model only QP is affected by block differences in design I and NQP in design II. Under the proportional model the confounded effect both in designs I and II is not precisely either QP or NQP but is a joint effect of QP and NQP.

2.2. *Choice of design*

$3 \times 2 \times 2$ qualitative-*cum*-quantitative experiment in 6 plot blocks should be carried out in at least two replications so as to provide adequate degrees of freedom (11 d.f. in this case) for performing reliable tests of significance. The two replications can be chosen in the following three ways:

(i) repeating design I in both the replications,

(ii) repeating design II in both the replications,

(iii) using designs I and II together.

From section 2.1. it is clear that the best choice for such types of experiments is choice (ii), *viz.*, repeating design II in both the replications.

2.3. *Analysis under the additive model—Design II repeated*

The additive model after simplification becomes:

$$\begin{aligned} y_{ijkl} = \mu &+ N_i + \delta Q_j + \delta(-1)^i (NQ)_j + P_k + (-1)^{1+k} (NP)_i \\ &+ \delta(-1)^{1+k} (QP)_j + \delta(-1)^{1+i+k} (NQP)_j + \gamma_l \\ &+ \beta_{ml} + e_{ijklm} \end{aligned}$$

where y_{ijkl} is the yield from the treatment combination $n_i q_j p_k$ in l-th replication and the other symbols have their usual meanings. The restrictions on the parameters are:

$$\delta = 0 \text{ for } i = 0, \text{ otherwise } \delta = 1$$

$$\left.\begin{array}{lll} \sum_{i=0}^{2} N_i = 0 & \sum_{k=0}^{1} P_k = 0 & \sum_{j=1}^{1} (NQP)_j = 0 \\ \sum_{j=0}^{1} Q_j = 0 & \sum_{i=0}^{2} (NP)_i = 0 & \sum_{l=1}^{2} \gamma_l = 0 \\ \sum_{j=0}^{1} (NQ)_j = 0 & \sum_{j=0}^{1} (QP)_j = 0 & \sum_{m=1}^{2} \beta_{ml} = 0 \\ & & \text{for } l = 1 \text{ and } 2. \end{array}\right\} \quad (10)$$

The normal equations for the components affected by block differences come out as;

$$\left.\begin{aligned} &8\,(NQP)_0+4\,[(NP)_2-(NP)_1]+2\,(\beta_{21}+\beta_{22}-\beta_{11}-\beta_{12}) \\ &\quad=(y_{201.}-y_{200.})-(y_{101.}-y_{100.}) \\ &8\,(NQP)_1+4\,[(NP)_2-(NP)_1]+2\,(\beta_{11}+\beta_{12}-\beta_{21}-\beta_{22}) \\ &\quad=(y_{211.}-y_{210.})-(y_{111.}-y_{110.}) \end{aligned}\right\} . \quad (11)$$

Normal equations for block parameters are:

$$\left.\begin{aligned} 6\mu+6\gamma_l+6\beta_{1l}+2\,[(NQP)_1-(NQP)_0]=B_{1l} \\ 6\mu+6\gamma_l+6\beta_{2l}+2\,[(NQP)_0-(NQP)_1]=B_{2l} \end{aligned}\right\} \quad (12)$$

for $l=1$ and 2.

Set (11) when solved with the help of set (12) gives:

$$\left[\widehat{(NQP)_1-(NQP)_0}\right]$$

$$=\frac{1}{8}\left\{\begin{aligned} &(y_{211.}-y_{210.})-(y_{111.}-y_{110.})-(y_{201.}-y_{200.}) \\ &\quad+(y_{101.}-y_{100.})-2\,[\text{'}n_0\text{ plots in }B_{11} \\ &\quad+B_{12}\text{'}-\text{'}n_0\text{ plots on }B_{21}+B_{22}\text{'}] \end{aligned}\right\} \quad (13)$$

where 'n_0 plots in $B_{11}+B_{12}$' stands for the total yield from those plots of first block of first replication and first block of second replication which receive 'n_0' dose of 'n'. Estimate given by (13) is NQP (1 d.f.) adjusted for blocks.

The sum of squares due to unconfounded effects can be obtained in the usual way. The sum of squares due to NQP partially confounded degree of freedom is

$$\frac{4}{3}\left[\widehat{(NQP)_1-(NQP)_0}\right]^2.$$

The relative information of NQP with respect to an unconfounded design is $\frac{1}{3}\sigma_{12}^2/\sigma_6^2$ where σ_6^2 and σ_{12}^2 are the variances per plot in case of designs with 6 plots and 12 plots per block respectively.

2.4. *Analysis under the proportional model—Design II repeated*

The proportional model after simplification becomes:

$$\begin{aligned} y_{ijkl} &= \mu+N_i+\delta Q_j+\delta'(-1)^i(NQ)_j+P_k+(-1)^{1+k}(NP)_i \\ &\quad+\delta(-1)^{1+k}(QP)_j+\delta'(-1)^{1+i+k}(NQP)_j+\gamma_l+\beta_{ml} \\ &\quad+e_{ijklm} \end{aligned}$$

where

$$\begin{array}{llll} \delta=0 & \text{for } i=0 & \delta'=0 & \text{for } i=0 \\ \delta=1 & \text{for } i=1 & \delta'=2 & \text{for } i=1 \\ \delta=2 & \text{for } i=2 & \delta'=1 & \text{for } i=2 \end{array}$$

and the other restrictions on the parameter are (10). Normal equations for the components affected by block differences come out as:

$$\left.\begin{aligned} &20\,(QP)_0+4\,(NP)_1+8\,(NP)_2+6\,(P_1-P_0)+(\beta_{22}+\beta_{21}-\beta_{11}-\beta_{12}) \\ &\qquad=(y_{101.}-y_{100.})+2\,(y_{201.}-y_{200.}) \\ &20\,(QP)_1+4\,(NP)_1+8\,(NP)_2+6\,(P_1-P_0)+(\beta_{11}+\beta_{12}-\beta_{22}-\beta_{21}) \\ &\qquad=(y_{111.}-y_{110.})+2\,(y_{211.}-y_{210.}) \end{aligned}\right\} \quad (14)$$

$$\left.\begin{aligned} &20\,(NQP)_0+4\,(NP)_2-8\,(NP)_1+2\,(P_0-P_1)+3\,(\beta_{22}+\beta_{21}-\beta_{11}-\beta_{12}) \\ &\qquad=(y_{201.}-y_{200.})-2\,(y_{101.}-y_{100.}) \\ &20\,(NQP)_1+4\,(NP)_2-8\,(NP)_1+2\,(P_0-P_1)+3\,(\beta_{11}+\beta_{12}-\beta_{22}-\beta_{21}) \\ &\qquad=(y_{211.}-y_{210.})-2\,(y_{111.}-y_{110.}) \end{aligned}\right\} \quad (15)$$

Normal equations for block parameters are:

$$\left.\begin{aligned} &6\mu+6\gamma_l+6\beta_{1l}+3\,[(NQP)_1-(NQP)_0]+[(QP)_1-(QP)_0]=B_{1l} \\ &6\mu+6\gamma_l+6\beta_{2l}+3\,[(NQP)_0-(NQP)_1]+[(QP)_0-(QP)_1]=B_{2l} \end{aligned}\right\} \quad (16)$$

for $l=1$ and 2.

set (14) gives:

$$20\,[(QP)_1-(QP)_0]+2\,(\beta_{11}+\beta_{12}-\beta_{21}-\beta_{22}) = \left\{\begin{aligned} &(y_{111.}-y_{110.})+2\,(y_{211.}-y_{210.}) \\ &\quad-(y_{101.}-y_{100.})-2\,(y_{201.}-y_{200.}) \end{aligned}\right\} \quad (17)$$

set (15) gives:

$$20\,[(NQP)_1-(NQP)_0]+6\,(\beta_{11}+\beta_{12}-\beta_{21}-\beta_{22}) = \left\{\begin{aligned} &(y_{211.}-y_{210.})-2\,(y_{111.}-y_{110.}) \\ &\quad-(y_{201.}-y_{200.})+2\,(y_{101.}-y_{100.}) \end{aligned}\right\}. \quad (18)$$

It is important to note that the estimates of $[(QP)_1-(QP_0)]$ and $[(NQP)_1-(NQP)_0]$, obtained by substituting the block parameter values from (16) in (17) and (18) would not be orthogonal. To overcome this difficulty some joint estimates of QP and NQP which are not only mutually orthogonal but orthogonal to all other effects are obtained as follows;

$3\times$ (17)—(18) gives:

$$\{3\,[(QP)_1-(QP)_0]-[(NQP)_1-(NQP)_0]\}$$
$$=\frac{1}{4}\left\{\begin{array}{l}(y_{111.}-y_{110.})+(y_{211.}-y_{210.})\\ \quad-(y_{101.}-y_{100.})-(y_{201.}-y_{200.})\end{array}\right\}. \tag{19}$$

The estimate given by (19) is ${}_{NQP}^{QP}\}$ 1 unconfounded degree of freedom. $3\times$ (18) + (17) when solved with the help of set (16) gives:

$$\{3\,[(NQP)_1-(NQP)_0]+[(QP)_1-(QP)_0]\}$$
$$=\frac{1}{4}\left\{\begin{array}{l}(y_{211.}-y_{210.})-(y_{111.}-y_{110.})-(y_{201.}-y_{200.})\\ \quad+(y_{101.}-y_{100.})-2\,[(\text{‘}n_0 \text{ plots in } B_{11}+B_{12}\text{’}\\ \quad-\text{‘}n_0 \text{ plots in } B_{21}+B_{22}\text{’}]\end{array}\right\}. \tag{20}$$

The estimate given by (20) is ${}_{NQP}^{QP}\}$ 1 partially confounded degree of freedom adjusted for blocks.

The sum of squares due to unconfounded effects can be obtained in the usual way. The sum of squares due to ${}_{NQP}^{QP}\}$ 1 unconfounded degree of freedom and ${}_{NQP}^{QP}\}$ 1 partially confounded degree of freedom are

$$\left\{3\,\widehat{[(QP)_1-(QP)_0]-[(NQP)_1-(NQP)_0]}\right\}^2$$

and

$$\frac{1}{3}\left\{3\,\widehat{[(NQP)_1-(NQP)_0)+[(QP)_1-(QP)_0]}\right\}^2$$

respectively.

The relative information of ${}_{NQP}^{QP}\}$ 1 partially confounded degree of freedom with respect to an unconfounded design is $\frac{1}{3}\,\sigma_{12}^2/\sigma_6^2$ where σ_6^2 and σ_{12}^2 are the variances per plot in case of designs with 6 plots and 12 plots per block respectively.

2.5. *Analysis when the non-zero levels are in the ratio* $1:\alpha$

The analysis under the additive model remains the same but the analysis under the proportional model will change as the weights 1 and 2 will change to 1 and α. δ and δ' defined for the proportional model will in this case have values:

$\delta = 0$ for $i = 0$ $\quad \delta' = 0$ for $i = 0$

$\delta = 1$ for $i = 1$ $\quad \delta' = \alpha$ for $i = 1$

$\delta = \alpha$ for $i = 2$ $\quad \delta' = 1$ for $i = 2$

With this change the analysis can be done on the same lines as in the case where the levels of 'n' are in the ratio 0 : 1 : 2. The component affected by block differences when design II is repeated in both the replications is

$$\{(\alpha + 1) [(NQP)_1 - (NQP)_0] + (\alpha - 1) [(QP)_1 - (QP)_0]\}.$$

3. 3×3×2 Qualitative-*cum*-Quantitative Experiments

In qualitative-*cum*-quantitative experiments there should be at least two non-zero quantities of the factor which is being tried in different forms. Thus the possible 3×3×2 qualitative-*cum*-quantitative experiments are of the type:

(i) 3 quantities of 'n', 3 qualities of 'n', 2 quantities or qualities of 'p'.

(ii) 3 quantities of 'n', 3 quantities or qualities of 'p', 2 qualities of 'n'.

It might be thought that in 6 plot blocks all the main effects in case of a qualitative-*cum*-quantitative experiment involving 3 quantities of 'n', 3 qualities of 'n', 2 quantities or qualities of 'p' can be kept free from confounding, but it can be shown that it is not so. With blocks of 6 plots, there will be three blocks in each complete replication and each block will contain 2 common treatments p_0 and p_1. The other 4 treatments in each block are to be chosen out of the remaining 12 treatments. If the quality main effect is to be kept free from confounding then there must be equal number of plots in every block receiving 'n' through each quality. But this is not possible as there are 4 plots and 3 qualities. Thus with block size 6 quality main effect cannot be kept free from confounding and also with block size 9 'p' main effect cannot be kept free from confounding. It is thus not advisible to use confounded design for experiments involving 3 quantities of 'n', 3 qualities of 'n', 2 quantities or qualities of 'p'.

Confounded design in 6 plot blocks for qualitative-*cum*-quantitative experiments involving 3 quantities of 'n', 3 quantities or qualities of 'p', 2 qualities of 'n' derivable from the classical 3×3×2 designs given in Kempthorne (1951) are presented in Appendix III.

3.1. *Confounded effects*

It can easily be seen that both in designs I and II, not only *NP* and *NQP* but *QP* is also affected by block differences. Thus the split *QP* (2 d.f.) and *NQP* (2 d.f.) does not help in seeing what has happened to this group of comparison. It can best be seen what has happened to this group of comparison by splitting these 4 degrees of freedom into two components, *viz*; one with 2 degrees of freedom obtained from the interaction *QP* at n_1 dose and the other with 2 degrees of freedom for the same interaction at n_2 dose of '*n*' as done in section *1.2*. An important advantage of this split of *QP* at n_1 and *QP* at n_2 is that the analysis based on this split under the proportional model is not very much different from that under the additive model. The analysis only differs in the sum of squares for *Q* and *NQ* which are free from bllck differences in both designs I and II. The components affected by block differences in design I and design II are:

Design No.	Components affected by block differences
I	(i) NP^2 (2 d.f.)
	(ii) $\left.\begin{matrix}QP\\NQP\end{matrix}\right\}$ 2 d.f.—which are the two independent comparisons between $(\lambda_0+\mu_1)$, $(\lambda_1+\mu_2)$, $(\lambda_2+\mu_0)$ where $\lambda_k = y_{11k} - y_{10k}$ and $\mu_k = y_{21k} - y_{20k}$.
II	(i) NP (2 d.f.)
	(ii) $\left.\begin{matrix}QP\\NQP\end{matrix}\right\}$ 2 d.f.—which are the two independent comparisons between $(\lambda_0+\mu_2)$, $(\lambda_1+\mu_0)$, $(\lambda_2+\mu_1)$.

It is evident from above that in both designs I and II, no information on *QP* at n_1 or *QP* at n_2 would be available. What would be available instead is a joint effect of *QP* and *NQP*. In case the experimenter is interested in the interaction *QP*, then neither of the designs I or II is suitable. *QP* at n_1 and *QP* at n_2 can both be kept totally free from confounding by adopting design III, given in Appendix IV, where NP^2 and *NP* are totally confounded in replications 1 and 2 respectively.

3.2. *Analysis under additive and proportional models—Design* III

The analysis for design III both under the additive and proportional models is simple because NP^2 and *NP* are totally confounded in replications 1 and 2 respectively. The sum of squares due to unconfounded effects can be calculated in the usual way and the sum of squares due to NP^2 and NP are to be calculated from replications 2 and 1 respectively.

In case the non-zero levels are in the ratio $1 : \alpha$, the analysis under additive model remains the same and for proportional model, weights 1 and α may be used instead of 1 and 2 in the calculation of the sum of squares due to Q and NQ.

4. 4×3×2. Qualitative-*cum*-Quantitative Experiments

Let the three factors be :—

(i) 4 qualities of 'n'—q_0, q_1, q_2, q_3,

(ii) 3 quantities of 'n'—n_0, n_1, n_2, in the ratio $0:1:2$

(iii) 2 quantities or qualities of 'p'—p_0, p_1.

Confounded 4×3×2 qualitative-*cum*-quantitative designs in 12 plot blocks are derivable from the classical 4×3×2 in 12 plot block designs given by Li (1944) by using dummy treatments where necessary. In case of qualitative-*cum*-quantitative experiments the usual 9 replications reduce to only 6 different ones because replications 2 and 3; 5 and 6; 8 and 9 become identical. These 6 distinct replications are given in Appendix V. These 6 replications could also be obtained otherwise by a method similar to the one used in section *1.1*. In what follows replications 1, 3, 5 will be referred to as designs I, II and III respectively and replications 2, 4, 6 as designs IV, V and VI respectively.

4.1. *Confounded effects*

The components affected by block differences in different designs are given in the table on next page.

It is evident from the table that the best choice for single replicate experiment would be any one of the designs IV to VI.

4.2. *Analysis for single replicate experiment—Additive model*:

The method of analysis for, say design VI, is indicated below and for any other choice the analysis can be done on similar lines.

The additive model after simplification becomes:

$$y_{ijk} = \mu + N_i + \delta Q_j + \delta (-1)^i (NQ)_j + P_k + (-1)^{1+k} (NP)_i + \delta (-1)^{1+k} (QP)_j + \delta (-1)^{1+i+k} (NQP)_j + \beta_m + e_{ijkm}$$

Design No.	Components affected by block differences	
	Under additive model	Under proportional model
I	QP [1 d.f.], the comparison $\{(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0\}$	$\left.\begin{matrix} QP \\ NQP \end{matrix}\right\}$ 1 d.f., the comparison $\left\{\begin{matrix} 3\,[(QP)_1+(QP)_3-(QP)_2-(QP)_0]-[(NQP)_1 \\ +(NQP)_3-(NQP)_2-(NQP)_0] \end{matrix}\right\}$
II	QP [1 d.f.], the comparison $\{(QP)_0 + (QP)_3 - (QP)_1 - (QP)_2\}$	$\left.\begin{matrix} QP \\ NQP \end{matrix}\right\}$ 1 d.f., the comparison $\left\{\begin{matrix} 3\,[(QP)_0+(QP)_3-(QP)_1-(QP)_2]-[(NQP_0)) \\ +(NQP)_3-(NQP)_1-(NQP)_2] \end{matrix}\right\}$
III	QP [1 d.f.], the comparison $\{(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3\}$	$\left.\begin{matrix} QP \\ NQP \end{matrix}\right\}$ 1 d.f., the comparison $\left\{\begin{matrix} 3\,[(QP)_0+(QP)_1-(QP)_2-(QP)_3]-[(NQP)_0 \\ +(NQP)_1-(NQP)_2-(NQP)_3) \end{matrix}\right\}$
IV	NQP [1 d.f.], the comparison $\{(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0\}$	$\left.\begin{matrix} QP \\ NQP \end{matrix}\right\}$ 1 d.f., the comparison $\left\{\begin{matrix} 3\,[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0] \\ + [(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0] \end{matrix}\right\}$
V	NQP [1 d.f.], the comparison $\{(NQP)_0 + (NQP)_3 - (NQP)_1 - (NQP)_2)\}$	$\left.\begin{matrix} QP \\ NQP \end{matrix}\right\}$ 1 d.f., the comparison $\left\{\begin{matrix} 3\,[(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1] \\ + [(QP)_0 + (QP)_3 - (QP)_2 - (QP)_1] \end{matrix}\right\}$
VI	NQP (1 d.f.), the comparison $\{(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3\}$	$\left.\begin{matrix} QP \\ NQP \end{matrix}\right\}$ 1 d.f., the comparison $\left\{\begin{matrix} 3\,[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3] \\ + [(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3] \end{matrix}\right\}$

where y_{ijk} is the yield from the treatment combination $n_i q_j p_k$ and the other symbols have their usual meanings. The restrictions on the parameters are:

$$\delta = 0 \text{ for } i = 0 \quad \text{otherwise } \delta = 1.$$

$$\left.\begin{array}{llll}\sum_{i=0}^{2} N_i = 0 & \sum_{j=0}^{3} (NQ)_j = 0 & \sum_{i=0}^{2} (NP)_i = 0 & \sum_{j=0}^{3} (NQP)_j = 0 \\ \sum_{j=0}^{3} Q_j = 0 & \sum_{k=0}^{1} P_k = 0 & \sum_{j=0}^{3} (QP)_j = 0 & \sum_{m=1}^{2} \beta_m = 0\end{array}\right\}. \quad (21)$$

The normal equations for the components affected by block differences come out as:

$$\left.\begin{array}{l}4(NQP)_0 + 2(\beta_1 - \beta_2) \\ \quad = (y_{201} - y_{200}) - (y_{101} - y_{100}) - \frac{1}{4}[(y_{2.1} - y_{2.0}) - (y_{1.1} - y_{1.0})] \\ 4(NQP)_1 + 2(\beta_1 - \beta_2) \\ \quad = (y_{211} - y_{210}) - (y_{111} - y_{110}) - \frac{1}{4}[(y_{2.1} - y_{2.0}) - (y_{1.1} - y_{1.0})] \\ 4(NQP)_2 + 2(\beta_2 - \beta_1) \\ \quad = (y_{221} - y_{220}) - (y_{121} - y_{120}) - \frac{1}{4}[(y_{2.1} - y_{2.0}) - (y_{1.1} - y_{1.0})] \\ 4(NQP)_3 + 2(\beta_2 - \beta_1) \\ \quad = (y_{231} - y_{230}) - (y_{131} - y_{130}) - \frac{1}{4}[(y_{2.1} - y_{2.0}) - (y_{1.1} - y_{1.0})]\end{array}\right\}. \quad (22)$$

Normal equations for block parameters are:

$$\left.\begin{array}{l}12\beta_m + 12\mu + 2(-1)^{1+m}[(NQP)_0 + (NQP)_1 - (NQP)_2 \\ \quad - (NQP)_3] = B_m \text{ for } m = 1, 2\end{array}\right\}. \quad (23)$$

Splitting the 3 degrees of freedom for NQP into three orthogonal components, *viz.*,

$$[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3];$$

$$[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]$$

and

$$[(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1]$$

each carrying one degree of freedom, it is clear from set (22) that the last two components are free from block differences and their estimates are:

$$\left[\widehat{(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1}\right]$$

$$= \frac{1}{4}\left\{\begin{array}{l}\sum_{i=1,2} \sum_{j=0,3} (-1)^i (y_{ij1} - y_{ij0}) \\ - \sum_{i=1,2} \sum_{j=1,2} (-1)^i (y_{ij1} - y_{ij0})\end{array}\right\}$$

$$\left[\widehat{(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0}\right]$$

$$= \frac{1}{4}\left\{\begin{array}{l} \sum\limits_{i=1,2} \sum\limits_{j=1,3} (-1)^i (y_{ij1} - y_{ij0}) \\ - \sum\limits_{i=1,2} \sum\limits_{j=0,2} (-1)^i (y_{ij1} - y_{ij0}) \end{array}\right\}.$$

But the comparison

$$[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3]$$

is affected by block differences and the corresponding normal equation when solved with the help of (23) gives:

$$\left\{\widehat{(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3}\right\}$$

$$= \frac{1}{4}\Big\{ \sum_{i=1,2} \sum_{j=0,1} (-1)^i (y_{ij1} - y_{ij0}) - \sum_{i=1,2} \sum_{j=2,3} (-1)^i$$

$$\times (y_{ij1} - y_{ij0}) - 2\,[\text{`}n_0 \text{ plots in } B_1\text{'} - \text{`}n_0 \text{ plots in } B_2\text{'}]\Big\}.$$

The sum of squares due to all the components other than *NQP* can be calculated in the usual way. The sum of squares due to unconfounded components of *NQP*, *viz.*,

$$[(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1]$$

and

$$[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]$$

are

$$\left[\widehat{(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1}\right]^2$$

and

$$\left[\widehat{(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0}\right]^2$$

respectively and the sum of squares due to partially confounded effect

$$[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3]$$

is

$$\frac{1}{3}\left[\widehat{(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3}\right]^2.$$

The relative information of *NQP* partially confounded degree of freedom with respect to an unconfounded design can easily be seen to be $\frac{1}{3}(\sigma_{24}^2/\sigma_{12}^2)$, where σ_{24}^2 and σ_{12}^2 are the variances per plot in case of designs with 24 plots and 12 plots per block respectively.

4.3. *Analysis for design* VI—*proportional model*

The proportional model after simplification becomes:

$$y_{ijk} = \mu + N_i + \delta Q_j + \delta' (-1)^i (NQ)_j + P_k + (-1)^{1+k} (NP)_i + \delta (-1)^{1+k} (QP)_j + \delta' (-1)^{1+i+k} (NQP)_j + \beta_m + e_{ijkm}$$

where

$\delta = 0$	for	$i = 0$	$\delta' = 0$	for	$i = 0$
$\delta = 1$	for	$i = 1$	$\delta' = 2$	for	$i = 1$
$\delta = 2$	for	$i = 2$	$\delta' = 1$	for	$i = 2$

and the other restrictions on the parameters are (21).

The normal equations for the components affected by block differences come out as:

$$\left.\begin{aligned}
&10(QP)_0 + (\beta_1 - \beta_2) \\
&\quad = (y_{101} - y_{100}) + 2(y_{201} - y_{200}) - \tfrac{1}{4}\{(y_{1.1} - y_{1.0}) + 2(y_{2.1} - y_{2.0})\} \\
&10(QP)_1 + (\beta_1 - \beta_2) \\
&\quad = (y_{111} - y_{110}) + 2(y_{211} - y_{210}) - \tfrac{1}{4}\{(y_{1.1} - y_{1.0}) + 2(y_{2.1} - y_{2.0})\} \\
&10(QP)_2 + (\beta_2 - \beta_1) \\
&\quad = (y_{121} - y_{120}) + 2(y_{221} - y_{220}) - \tfrac{1}{4}\{(y_{1.1} - y_{1.0}) + 2(y_{2.1} - y_{2.0})\} \\
&10(QP)_3 + (\beta_2 - \beta_1) \\
&\quad = (y_{131} - y_{130}) + 2(y_{231} - y_{230}) - \tfrac{1}{4}\{(y_{1.1} - y_{1.0}) + 2(y_{2.1} - y_{2.0})\}
\end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned}
&10(NQP)_0 + 3(\beta_1 - \beta_2) \\
&\quad = (y_{201} - y_{200}) - 2(y_{101} - y_{100}) - \tfrac{1}{4}\{(y_{2.1} - y_{2.0}) - 2(y_{1.1} - y_{1.0})\} \\
&10(NQP)_1 + 3(\beta_1 - \beta_2) \\
&\quad = (y_{211} - y_{210}) - 2(y_{111} - y_{110}) - \tfrac{1}{4}\{(y_{2.1} - y_{2.0}) - 2(y_{1.1} - y_{1.0})\} \\
&10(NQP)_2 + 3(\beta_2 - \beta_1) \\
&\quad = (y_{221} - y_{220}) - 2(y_{121} - y_{120}) - \tfrac{1}{4}\{(y_{2.1} - y_{2.0}) - 2(y_{1.1} - y_{1.0})\} \\
&10(NQP)_3 + 3(\beta_2 - \beta_1) \\
&\quad = (y_{231} - y_{230}) - 2(y_{131} - y_{130}) - \tfrac{1}{4}\{(y_{2.1} - y_{2.0}) - 2(y_{1.1} - y_{1.0})\}
\end{aligned}\right\} \quad (25)$$

Normal equations for block parameters are:

$$\begin{aligned}
12\mu + 12\beta_m + (-1)^{1+m}\{[(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3] \\
+ 3[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3]\} \\
= B_m \text{ for } m = 1, 2.
\end{aligned} \quad (26)$$

It is clear from the normal equations that both QP and NQP are affected by block differences. The 3 degrees of freedom for QP can be split up into three orthogonal components, *viz.*,

$$\{(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3\},$$
$$\{(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0\}$$

and

$$\{(QP)_0 + (QP)_3 - (QP)_1 - (QP)_2\}$$

each carrying one degree of freedom. From set of normal equations (24), it is clear that the last two components are free from block differences and their estimates are:

$$\left[\widehat{(QP)_0 + (QP)_3 - (QP)_2 - (QP)_1}\right]$$
$$= \frac{1}{10}\left\{ \sum_{j=0,\,3} [(y_{1j1} - y_{1j0}) + 2\,(y_{2j1} - y_{2j0})] - \sum_{j=1,\,2} [(y_{1j1} - y_{1j0}) + 2\,(y_{2j1} - y_{2j0})]\right\}$$

$$\left[\widehat{(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0}\right]$$
$$= \frac{1}{10}\left\{ \sum_{j=1,\,3} [(y_{1j1} - y_{1j0}) + 2\,(y_{2j1} - y_{2j0})] - \sum_{j=0,\,2} [(y_{1j1} - y_{1j0}) + 2\,(y_{2j1} - y_{2j0})]\right\}.$$

But the component

$$[(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3]$$

is affected by block differences and the corresponding normal equation is:

$$10\,[(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3] + 4\,(\beta_1 - \beta_2)$$
$$= \sum_{j=0,\,1} [(y_{1j1} - y_{1j0}) + 2\,(y_{2j1} - y_{2j0})] - \sum_{j=2,\,3} [(y_{1j1} - y_{1j0}) + 2\,(y_{2j1} - y_{2j0})]. \qquad (27)$$

Similarly splitting NQP (3 d.f.) into three orthogonal components, *viz.*,

$$[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3];$$
$$[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]$$

and

$$[(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1]$$

each carrying one degree of freedom. It is clear that the last two components are free from block differences and their estimates are:

$$\left[\widehat{(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_0}\right]$$

$$= \frac{1}{10}\left\{ \sum_{j=0,\,3} [(y_{2j1} - y_{2j0}) - 2\,(y_{1j1} - y_{1j0})] - \sum_{j=1,\,2} [(y_{2j1} - y_{2j0}) - 2\,(y_{1j1} - y_{1j0})]\right\}$$

$$\left[\widehat{(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0}\right]$$

$$= \frac{1}{10}\left\{ \sum_{j=1,\,3} [(y_{2j1} - y_{2j0}) - 2\,(y_{1j1} - y_{1j0})] - \sum_{j=0,\,2} [(y_{2j1} - y_{2j0}) - 2\,(y_{1j1} - y_{1j0})]\right\}.$$

But the component

$$[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3]$$

is affected by block differences and the corresponding normal equation is:

$$10\,[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3] + 12\,(\beta_1 - \beta_2)$$

$$= \left\{ \sum_{j=0,\,1} [(y_{2j1} - y_{2j0}) - 2\,(y_{1j1} - y_{1j0})] - \sum_{j=2,\,3} [(y_{2j1} - y_{2j0}) - 2\,(y_{1j1} - y_{1j0})]\right\}. \tag{28}$$

It is important to note that if the block parameter values are substituted from set (26) in (27) and (28) and the resultant equations solved, the estimates of

$$[(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3]$$

and

$$[(NQP_0) + (NQP)_1 - (NQP)_2 - (NQP)_3]$$

so obtained would not be orthogonal. To overcome this difficulty, some joint estimates of

$$[(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3]$$

and

$$[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3]$$

which are not only mutually orthogonal but orthogonal to all other effects are obtained as follows:

$3 \times (27) - (28)$ gives:

$$\Big\{3\,[(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3] - [(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3]\Big\}$$

$$= \frac{1}{2}\Big\{\sum_{i=1,2}\ \sum_{j=0,1}(y_{ij1} - y_{ij0}) - \sum_{i=1,2}\ \sum_{j=2,3}(y_{ij1} - y_{ij0})\Big\}. \quad (29)$$

Estimate given by (29) is $\left.\begin{smallmatrix}QP\\NQP\end{smallmatrix}\right\}$ 1 unconfounded degree of freedom.

$3 \times (28) + (27)$ when solved with the help of set (26) gives:

$$\Big\{3\,[(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3] + [(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3]\Big\}$$

$$= \frac{1}{2}\left\{\begin{array}{l}\sum_{i=1,2}\ \sum_{j=0,1}(-1)^i\,[y_{ij1} - y_{ij0}] - \sum_{i=1,2}\ \sum_{j=2,3}(-1)^i\,[y_{ij1} - y_{ij0}] \\ \qquad - 2\,[\text{'}n_0 \text{ plots in } B_1\text{'} - \text{'}n_0 \text{ plots in } B_2\text{'}]\end{array}\right\}. \quad (30)$$

The estimate given by (30) is $\left.\begin{smallmatrix}QP\\NQP\end{smallmatrix}\right\}$ partially confounded degree of freedom.

The sum of squares due to all the components other than QP and NQP can be calculated in the usual way. The sum of squares due to unconfounded components of QP and NQP are:

$$\frac{5}{2}\left[\widehat{(QP)_0 + (QP)_3 - (QP)_2 - (QP)_1}\right]^2 + \frac{5}{2}\left[\widehat{(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0}\right]^2$$

and

$$\frac{5}{2}\left[\overline{(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1}\right]^2$$

$$+\frac{5}{2}\left[\overline{(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0}\right]^2$$

respectively. The sum of squares due to $\left.{}^{QP}_{NQP}\right\}$1 unconfounded degree of freedom and $\left.{}^{QP}_{NQP}\right\}$ 1 partially confounded degree of freedom are

$$\frac{1}{4}\left\{3\ [(QP)_0 + \overline{(QP)_1 - (QP)_2 - (QP)_3] - [(NQP)_0 + (NQP)_1} - (NQP)_2 - (NQP)_3]\right\}^2$$

$$+\frac{1}{12}\left\{3\ [(NQP)_0 + \overline{(NQP)_1 - (NQP)_2 - (NQP)_3 + [(QP)_0 + (QP)_1} - (QP)_2 - (QP)_3]\right\}^2 \text{ respectively.}$$

The relative information of $\left.{}^{QP}_{NQP}\right\}$ partially confounded degree of freedom with respect to an unconfounded design can easily be seen to be $\frac{1}{3}(\sigma_{24}{}^2/\sigma_{12}{}^2)$.

4.4. *Choice for two replicate design*

Design for two replications can be chosen in the following alternative ways:

(i) choosing any one of the six possible designs preferably from designs IV to VI and using the same in both the replications;

(ii) choosing two different designs for the two replications. The two different designs can be chosen in the following ways:

(*a*) choosing any two designs from I to III,

(*b*) choosing any two designs from IV to VI;

(*c*) choosing one from I to III and the second the corresponding number from IV to VI, *viz*., I and IV; II and V; III and VI;

(*d*) choices of the type I and V; I and VI; II and IV; II and VI; III and IV; III and V.

With the help of section 4.1 it can easily be seen that out of he choices (*a*) to (*d*), choice (*c*) is the best because with this choice more information is available on QP and NQP separately rather than $QP + NQP$. Choice (*c*) has been referred to by Fisher (1935) as a modification of choice (i) of repeating the same design in both the replications.

4.5. *Analysis for two replicate design*

Analysis for choice (i) is simple as the same degree of freedom is partially affected by block differences in both the replications.

The method of analysis for choice (*c*), say for designs I and IV, is indicated below and for any other choice the analysis can be done on similar lines.

Proportional model.—The normal equations for the components affected by block differences come out as:

$$\left.\begin{aligned}
&20\,(QP)_0 + 3\,(\beta_{11} - \beta_{21}) + (\beta_{12} - \beta_{22}) \\
&\quad = 2\,(y_{201.} - y_{200.}) + (y_{101.} - y_{100.}) - \tfrac{1}{4}\,[2\,(y_{2.1.} - y_{2.0.}) + (y_{1.1.} - y_{1.0.})] \\
&20\,(QP)_1 + 3\,(\beta_{21} - \beta_{11}) + (\beta_{22} - \beta_{12}) \\
&\quad = 2\,(y_{211.} - y_{210.}) + (y_{111.} - y_{110.}) - \tfrac{1}{4}\,[2\,(y_{2.1.} - y_{2.0.}) + (y_{1.1.} - y_{1.0.})] \\
&20\,(QP)_2 + 3\,(\beta_{11} - \beta_{21}) + (\beta_{12} - \beta_{22}) \\
&\quad = 2\,(y_{221.} - y_{220.}) + (y_{121.} - y_{120.}) - \tfrac{1}{4}\,[2\,(y_{2.1.} - y_{2.0.}) + (y_{1.1.} - y_{1.0.})] \\
&20\,(QP)_3 + 3\,(\beta_{21} - \beta_{11}) + (\beta_{22} - \beta_{12}) \\
&\quad = 2\,(y_{231.} - y_{230.}) + (y_{131.} - y_{130.}) - \tfrac{1}{4}\,[2\,(y_{2.1.} - y_{2.0.}) + (y_{1.1.} - y_{1.0.})]
\end{aligned}\right\} \quad (31)$$

$$\left.\begin{aligned}
&20\,(NQP)_0 + (\beta_{21} - \beta_{11}) + 3\,(\beta_{12} - \beta_{22}) \\
&\quad = (y_{201.} - y_{200.}) - 2\,(y_{101.} - y_{100.}) - \tfrac{1}{4}\,[(y_{2.1.} - y_{2.0.}) - 2\,(y_{1.1.} - y_{1.0.})] \\
&20\,(NQP)_1 + (\beta_{11} - \beta_{21}) + 3\,(\beta_{22} - \beta_{12}) \\
&\quad = (y_{211.} - y_{210.}) - 2\,(y_{111.} - y_{110.}) - \tfrac{1}{4}\,[(y_{2.1.} - y_{2.0.}) - 2\,(y_{1.1.} - y_{1.0.})] \\
&20\,(NQP)_2 + (\beta_{21} - \beta_{11}) + 3\,(\beta_{12} - \beta_{22}) \\
&\quad = (y_{221.} - y_{220.}) - 2\,(y_{121.} - y_{120.}) - \tfrac{1}{4}\,[(y_{2.1.} - y_{2.0.}) - 2\,(y_{1.1.} - y_{1.0.})] \\
&20\,(NQP)_3 + (\beta_{11} - \beta_{21}) + 3\,(\beta_{22} - \beta_{12}) \\
&\quad = (y_{231.} - y_{230.}) - 2\,(y_{131.} - y_{130.}) - \tfrac{1}{4}\,[(y_{2.1.} - y_{2.0.}) - 2\,(y_{1.1.} - y_{1.0.})]
\end{aligned}\right\} \quad (32)$$

Normal equations for block parameters are:

$$\left.\begin{aligned}
&12\beta_{m1} + 12\mu + 12\gamma_1 + 3\,(-1)^{1+m}\,[(QP)_0 + (QP)_2 - (QP)_1 - (QP)_3] \\
&\quad + (-1)^{1+m}\,[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0] = B_{m1} \\
&12\beta_{m2} + 12\mu + 12\gamma_2 + 3\,(-1)^{1+m}\,[(QP)_0 + (QP)_2 - (QP)_1 - (QP)_3] \\
&\quad + 3\,(-1)^{1+m}\,[(NQP)_0 + (NQP)_2 - (NQP)_1 - (NQP)_3] = B_{m2} \\
&\qquad\qquad \text{for } m = 1 \text{ and } 2
\end{aligned}\right\} . \quad (33)$$

Splitting QP (3 d.f.) into orthogonal components each carrying one degree of freedom we have:

$$\left[\overbrace{(QP)_0 + (QP)_1 - (QP)_2 - (QP)_3}\right]$$

$$= \frac{1}{20} \left\{ \begin{array}{l} \sum\limits_{j=0,1} [2(y_{2j1.} - y_{2j0.}) + (y_{1j1.} - y_{1j0.})] \\ \qquad - \sum\limits_{j=2,3} [2(y_{2j1.} - y_{2j0.}) + (y_{1j1.} - y_{1j0.})] \end{array} \right\}$$

$$\left[\overbrace{(QP)_0 + (QP)_3 - (QP)_2 - (QP)_1}\right]$$

$$= \frac{1}{20} \left\{ \begin{array}{l} \sum\limits_{j=0,3} [2(y_{2j1.} - y_{2j0.}) + (y_{1j1.} - y_{1j0.})] \\ \qquad - \sum\limits_{j=1,2} [2(y_{2j1.} - y_{2j0.}) + (y_{1j1.} - y_{1j0.})] \end{array} \right\}$$

$$20\,[(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0] + 12\,(\beta_{21} - \beta_{11}) + 4\,(\beta_{22} - \beta_{12})$$

$$= \left\{ \begin{array}{l} \sum\limits_{j=1,3} [2(y_{2j1.} - y_{2j0.}) + (y_{1j1.} - y_{1j0.})] \\ \qquad - \sum\limits_{j=0,2} [2(y_{2j1.} - y_{2j0.}) + (y_{1j1.} - y_{1j0.})] \end{array} \right\}. \tag{34}$$

Similarly splitting NQP (3 d.f.) into orthogonal components each carrying one degree of freedom we have:

$$\left[\overbrace{(NQP)_0 + (NQP)_1 - (NQP)_2 - (NQP)_3}\right]$$

$$= \frac{1}{20} \left\{ \sum_{j=0,1} [(y_{2j1.} - y_{2j0.}) - 2(y_{1j1.} - y_{1j0.})] \right.$$

$$\left. - \sum_{j=2,3} [(y_{2j1.} - y_{2j0.}) - 2(y_{1j1.} - y_{1j0.})] \right\}$$

$$\left[\overbrace{(NQP)_0 + (NQP)_3 - (NQP)_2 - (NQP)_1}\right]$$

$$= \frac{1}{20} \left\{ \sum_{j=0,3} [(y_{2j1.} - y_{2j0.}) - 2(y_{1j1.} - y_{1j0.})] \right.$$

$$\left. - \sum_{j=2,1} [(y_{2j1.} - y_{2j0.}) - 2(y_{1j1.} - y_{1j0.})] \right\}$$

$$20\,[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0] + 4\,(\beta_{11} - \beta_{21}) + 12\,(\beta_{22} - \beta_{12})$$

$$= \Big\{ \sum_{j=1,3} [(y_{2j1.} - y_{2j0.}) - 2\,(y_{1j1.} - y_{1j0.})] - \sum_{j=0,2} [(y_{2j1.} - y_{2j0.}) - 2\,(y_{1j1.} - y_{1j0.})] \Big\}. \quad (35)$$

It is important to note that if the block parameter values are substituted in (34) and (35), the resulting equations give estimates of

$$[(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0]$$

and

$$[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]$$

which are not orthogonal. To overcome this difficulty, joint estimates of

$$[(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0]$$

and

$$[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]$$

which are not only mutually orthogonal but orthogonal to all other effects are obtained as follows:—

$3 \times (35) + (34)$ when solved with the help of (33) gives:

$$\Big\{[(QP)_1+(QP)_3-(QP)_2-(QP)_0] + 3\,[(NQP)_1+(NQP)_3-(NQP)_2-(NQP)_0]\Big\}$$

$$= \Big\{3\Big[\sum_{i=1,2}\sum_{j=1,3}(-1)^i(y_{ij1.} - y_{ij0.}) - \sum_{i=1,2}\sum_{j=0,2}(-1)^i(y_{ij1.} - y_{ij0.})\Big] - 2\,(B_{22} - B_{12})\Big\}. \quad (36)$$

Also $3 \times (34) - (35)$ when solved with the help of (33) gives:

$$\{3\,[(QP)_1+(QP)_3-(QP)_2-(QP)_0] - [(NQP)_1+(NQP)_3-(NQP)_2-(NQP)_0]\}$$

$$= \Big\{3\Big[\sum_{i=1,2}\sum_{j=1,3}(y_{ij1.}-y_{ij0.}) + \sum_{i=1,2}\sum_{j=0,2}(y_{ij1.}-y_{ij0.})\Big] - 2\,[B_{21} - B_{11}]\Big\}. \quad (37)$$

Estimates given by (36) and (37) are $\left.\begin{matrix}QP\\NQP\end{matrix}\right\}$ 2 partially confounded degrees of freedom adjusted for blocks.

The sum of squares due to unconfounded effects can be obtained in the usual way. The sum of squares due to partially confounded effects

$$\{[(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0] + 3\,[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]\}$$

and

$$\{3\,[(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0] - [(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]\}$$

are

$$\frac{1}{192}\left\{3\left[\sum_{i=1,2}\sum_{j=1,3}(-1)^i(y_{ij1.}-y_{ij0.}) - \sum_{i=1,2}\sum_{j=0,2}(-1)^i(y_{ij1.}-y_{ij0.})\right] - 2\,(B_{22}-B_{12})\right\}^2$$

and

$$\frac{1}{192}\left\{3\left[\sum_{i=1,2}\sum_{j=1,3}(y_{ij1.}-y_{ij0.}) - \sum_{i=1,2}\sum_{j=0,2}(y_{ij1.}-y_{ij0.})\right] - 2\,(B_{21}-B_{11})\right\}^2$$

respecitvely.

Additive model.—Under the additive model, the components

$$[(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0]$$

of QP and

$$[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]$$

of NQP are partially affected by block differences in replications 1 (design I) and 2 (design IV) respectively. The sum of squares due to these partially affected components of QP and NQP are the same as that for

$$\{3\,[(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0] - [(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0]\}$$

and

$$\{3\,[(NQP)_1 + (NQP)_3 - (NQP)_2 - (NQP)_0] + [(QP)_1 + (QP)_3 - (QP)_2 - (QP)_0]\}$$

respectively under the proportional model.

SUMMARY

Confounding in qualitative-*cum*-quantitative experiments involving dummy treatments and their analyses present some novel features not met with in the analysis of ordinary factorial experiments. These features have been studied in detail by discussing the possible types of confounding and presenting the methods of analysis, both under the additive and proportional models for the following types of commonly used symmetrical and asymmetrical designs:

'n'		'p'
Quantities	Qualities	Quantities or qualities
3	3	3
3	2	2
3	3	2
3	2	3
3	4	2

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APPENDIX I

Designs for 3³ qualitative-cum-quantitative experiments in 9 plot blocks

Design 1									Design 2								
B_1			B_2			B_3			B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	1	1	1	2	1	1	0	1	1	1	1	1	2	1	1	0	1
1	2	2	1	0	2	1	1	2	1	2	2	1	0	2	1	1	2
2	0	0	2	1	0	2	2	0	2	1	0	2	2	0	2	0	0
2	1	1	2	2	1	2	0	1	2	2	1	2	0	1	2	1	1
2	2	2	2	0	2	2	1	2	2	0	2	2	1	2	2	2	2

OR			*OR*		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
I_0	I_1	I_2	I_0	I_1	I_2
I_0'	I_1'	I_2'	I_1'	I_2'	I_0'

Design 3									*Design* 4								
B_1			B_2			B_3			B_1			B_2			B_3		
n	q	p	n	q	p	n	q	p	n	q	p	n	q	p	n	q	p
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	1	1	1	2	1	1	0	1	1	1	1	1	2	1	1	0	1
1	2	2	1	0	2	1	1	2	1	2	2	1	0	2	1	1	2
2	2	0	2	0	0	2	1	0	2	0	0	2	1	0	2	2	0
2	0	1	2	1	1	2	2	1	2	2	1	2	0	1	2	1	1
2	1	2	2	2	2	2	0	2	2	1	2	2	2	2	2	0	2

OR			*OR*		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
I_0	I_1	I_2	I_0	I_1	I_2
I_2'	I_0'	I_1'	J_0'	J_1'	J_2'

Design 5									*Design 6*								
B_1			B_2			B_3			B_1			B_2			B_3		
n	q	p	n	q	p	n	q	p	n	q	p	n	q	p	n	q	p
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	1	1	1	2	1	1	0	1	1	1	1	1	2	1	1	0	1
1	2	2	1	0	2	1	1	2	1	2	2	1	0	2	1	1	2
2	1	0	2	2	0	2	0	0	2	2	0	2	0	0	2	1	0
2	0	1	2	1	1	2	2	1	2	1	1	2	2	1	2	0	1
2	2	2	2	0	2	2	1	2	2	0	2	2	1	2	2	2	2

OR			*OR*		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
I_0	I_1	I_2	I_0	I_1	I_2
J_1'	J_2'	J_0'	J_2'	J_0'	J_1'

Design 7									Design 8								
B_1			B_2			B_3			B_1			B_2			B_3		
n	q	p	n	q	p	n	q	p	n	q	p	n	q	p	n	q	p
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	2	1	1	0	1	1	1	1	1	2	1	1	0	1	1	1	1
1	1	2	1	2	2	1	0	2	1	1	2	1	2	2	1	0	2
2	0	0	2	1	0	2	2	0	2	1	0	2	2	0	2	0	0
2	2	1	2	0	1	2	1	1	2	0	1	2	1	1	2	2	1
2	1	2	2	2	2	2	0	2	2	2	2	2	0	2	2	1	2

OR			OR		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
J_0	J_1	J_2	J_0	J_1	J_2
J_0'	J_1'	J'_2	J_1'	J_2'	J_0'

Design 9									Design 10								
B_1			B_2			B_3			B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	2	1	1	0	1	1	1	1	1	2	1	1	0	1	1	1	1
1	1	2	1	2	2	1	0	2	1	1	2	1	2	2	1	0	2
2	2	0	2	0	0	2	1	0	2	0	0	2	1	0	2	2	0
2	1	1	2	2	1	2	0	1	2	1	1	2	2	1	2	0	1
2	0	2	2	1	2	2	2	2	2	2	2	2	0	2	2	1	2

OR			OR		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
J_0	J_1	J_2	J_0	J_1	J_2
J_2'	J_0'	J_1'	I_0'	I'_1	I_2'

Design 11									*Design* 12								
B_1			B_2			B_3			B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	2	1	1	0	1	1	1	1	1	2	1	1	0	1	1	1	1
1	1	2	1	2	2	1	0	2	1	1	2	1	2	2	1	0	2
2	1	0	2	2	0	2	0	0	2	2	0	2	0	0	2	1	0
2	2	1	2	0	1	2	1	1	2	0	1	2	1	1	2	2	1
2	0	2	2	1	2	2	2	2	2	1	2	2	2	2	2	0	2

OR			*OR*		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
J_0	J_1	J_2	J_0	J_1	J_2
I_1'	I_2'	I_0'	I_2'	I_0'	I_1'

Design 13

B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0
1	1	1	1	2	1	1	0	1
1	2	2	1	0	2	1	1	2
2	0	0	2	2	0	2	1	0
2	1	1	2	0	1	2	2	1
2	2	2	2	1	2	2	0	2

OR

p_0	p_0	p_0
p_1	p_1	p_1
p_2	p_2	p_2
I_0	I_1	I_2
I_0'	I_2'	I_1'

Design 14

B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0
1	1	1	1	2	1	1	0	1
1	2	2	1	0	2	1	1	2
2	1	0	2	0	0	2	2	0
2	2	1	2	1	1	2	0	1
2	0	2	2	2	2	2	1	2

OR

p_0	p_0	p_0
p_1	p_1	p_1
p_2	p_2	p_2
I_0	I_1	I_2
I_1'	I_0'	I_2'

Design 15									*Design* 16								
B_1			B_2			B_3			B_1			B_2			B_3		
n	q	p	n	q	p	n	q	p	n	q	p	n	q	p	n	q	p
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	1	1	1	2	1	1	0	1	1	1	1	1	2	1	1	0	1
1	2	2	1	0	2	1	1	2	1	2	2	1	0	2	1	1	2
2	2	0	2	1	0	2	0	0	2	0	0	2	2	0	2	1	0
2	0	1	2	2	1	2	1	1	2	2	1	2	1	1	2	0	1
2	1	2	2	0	2	2	2	2	2	1	2	2	0	2	2	2	2

OR			*OR*		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
I_0	I_1	I_2	I_0	I_1	I_2
I_2'	I_1'	I_0'	J_0'	J_2'	J_1'

Design 17									*Design* 18								
B_1			B_2			B_3			B_1			B_2			B_3		
n	q	p	n	q	p	n	q	p	n	q	p	n	q	p	n	q	p
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	1	1	1	2	1	1	0	1	1	1	1	1	2	1	1	0	1
1	2	2	1	0	2	1	1	2	1	2	2	1	0	2	1	1	2
2	1	0	2	0	0	2	2	0	2	2	0	2	1	0	2	0	0
2	0	1	2	2	1	2	1	1	2	1	1	2	0	1	2	2	1
2	2	2	2	1	2	2	0	2	2	0	2	2	2	2	2	1	2

OR (Design 17)

p_0	p_0	p_0
p_1	p_1	p_1
p_2	p_2	p_2
I_0	I_1	I_2
J_1'	J_0'	J_2'

OR (Design 18)

p_0	p_0	p_0
p_1	p_1	p_1
p_2	p_2	p_2
I_0	I_1	I_2
J_2'	J_1'	J_0'

Design 19									Design 20								
B_1			B_2			B_3			B_1			B_2			B_3		
n	q	p	n	q	p	n	q	p	n	q	p	n	q	p	n	q	p
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	2	1	1	0	1	1	1	1	1	2	1	1	0	1	1	1	1
1	1	2	1	2	2	1	0	2	1	1	2	1	2	2	1	0	2
2	0	0	2	2	0	2	1	0	2	1	0	2	0	0	2	2	0
2	2	1	2	1	1	2	0	1	2	0	1	2	2	1	2	1	1
2	1	2	2	0	2	2	2	2	2	2	2	2	1	2	2	0	2

OR (Design 19)			OR (Design 20)		
p_0	p_0	p_0	p_0	p_0	p_0
p_1	p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_2	p_2	p_2	p_2
J_0	J_1	J_2	J_0	J_1	J_2
J_0'	J_2'	J'_1	J_1'	J_0'	J_2'

Design 21									Design 22								
B_1			B_2			B_3			B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	2	1	1	0	1	1	1	1	1	2	1	1	0	1	1	1	1
1	1	2	1	2	2	1	0	2	1	1	2	1	2	2	1	0	2
2	2	0	2	1	0	2	0	0	2	0	0	2	2	0	2	1	0
2	1	1	2	0	1	2	2	1	2	1	1	2	0	1	2	2	1
2	0	2	2	2	2	2	1	2	2	2	2	2	1	2	2	0	2

OR (Design 21)

p_0	p_0	p_0
p_1	p_1	p_1
p_2	p_2	p_2
J_0	J_1	J_2
J_2'	J_1'	J_0'

OR (Design 22)

p_0	p_0	p_0
p_1	p_1	p_1
p_2	p_2	p_2
J_0	J_1	J_2
I_0'	I_2'	I_1'

Design 23									*Design* 24								
B_1			B_2			B_3			B_1			B_2			B_3		
n	q	p	n	q	p	n	q	p	n	q	p	n	q	p	n	q	p
0	–	0	0	–	0	0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1	0	–	1	0	–	1
0	–	2	0	–	2	0	–	2	0	–	2	0	–	2	0	–	2
1	0	0	1	1	0	1	2	0	1	0	0	1	1	0	1	2	0
1	2	1	1	0	1	1	1	1	1	2	1	1	0	1	1	1	1
1	1	2	1	2	2	1	0	2	1	1	2	1	2	2	1	0	2
2	1	0	2	0	0	2	2	0	2	2	0	2	1	0	2	0	0
2	2	1	2	1	1	2	0	1	2	0	1	2	2	1	2	1	1
2	0	2	2	2	2	2	1	2	2	1	2	2	0	2	2	2	2

OR (Design 23) and *OR* (Design 24):

p_0	p_1	p_2	p_0	p_1	p_2
p_0	p_1	p_2	p_0	p_1	p_2
p_0	p_1	p_2	p_0	p_1	p_2
J_0	J_1	J_2	J_0	J_1	J_2
I'_1	I_0'	I'_2	I_2'	I_1'	I_0'

APPENDIX II

*Designs for 3×2×2 qualitative-*cum*-quantitative experiments in 6 plot blocks*

Replication 1 or Design I						Replication 2 or Design II					
B_1			B_2			B_1			B_2		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1
1	0	0	1	0	1	1	0	1	1	0	0
1	1	1	1	1	0	1	1	0	1	1	1
2	0	0	2	0	1	2	0	0	2	0	1
2	1	1	2	1	0	2	1	1	2	1	0

Appendix III

Designs for qualitative-cum-quantitative experiments involving 3 *levels of* 'n', 3 *levels of* 'p', 2 *qualities of* 'n' *in* 6 *plot blocks*

Design I

Replication 1									*Replication* 2								
B_1			B_2			B_3			B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	1	0	–	0	0	–	0	0	–	1	0	–	0	0	–	2
0	–	2	0	–	1	0	–	2	0	–	2	0	–	1	0	–	0
1	0	0	1	0	2	1	0	1	1	0	2	1	0	1	1	0	0
1	1	2	1	1	1	1	1	0	1	1	0	1	1	2	1	1	1
2	0	1	2	0	0	2	0	2	2	0	0	2	0	2	2	0	1
2	1	0	2	1	2	2	1	1	2	1	1	2	1	0	2	1	2

Design II

Replication 1									*Replication* 2								
B_1			B_2			B_3			B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	1	0	–	0	0	–	0	0	–	2	0	–	0	0	–	1
0	–	2	0	–	2	0	–	1	0	–	0	0	–	2	0	–	0
1	0	0	1	0	1	1	0	2	1	0	1	1	0	2	1	0	0
1	1	1	1	1	2	1	1	0	1	1	0	1	1	1	1	1	2
2	0	2	2	0	0	2	0	1	2	0	0	2	0	1	2	1	1
2	1	0	2	1	1	2	1	2	2	1	2	2	1	0	2	0	2

Appendix IV

Design III

Replication 1									*Replication 2*								
B_1			B_2			B_3			B_1			B_2			B_3		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	1	0	–	2	0	–	0	0	–	1	0	–	2
0	–	0	0	–	1	0	–	2	0	–	0	0	–	1	0	–	2
1	0	1	1	0	2	1	0	0	1	0	2	1	0	0	1	0	1
1	1	1	1	1	2	1	1	0	1	1	2	1	1	0	1	1	1
2	0	2	2	0	0	2	0	1	2	0	1	2	0	2	2	0	0
2	1	2	2	1	0	2	1	1	2	1	1	2	1	2	2	1	0

Appendix V

Designs for 4×3×2 *qualitative*-cum-*quantative experiments in* 12 *plot blocks*

Replication 1 or Design I						Replication 2 or Design IV					
B_1			B_2			B_1			B_2		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0
0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1
0	–	1	0	–	1	0	–	1	0	–	1
1	0	1	1	1	1	1	0	0	1	1	0
1	2	1	1	3	1	1	2	0	1	3	0
1	1	0	1	2	0	1	1	1	1	0	1
1	3	0	1	0	0	1	3	1	1	2	1
2	0	1	2	1	1	2	0	1	2	1	1
2	2	1	2	3	1	2	2	1	2	3	1
2	1	0	2	2	0	2	1	0	2	0	0
2	3	0	2	0	0	2	3	0	2	2	0

Replication 3 *or* *Design II*						*Replication* 4 *or* *Design V*					
B_1			B_2			B_1			B_2		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0
0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1
0	–	1	0	–	1	0	–	1	0	–	1
1	0	1	1	1	1	1	0	0	1	1	0
1	3	1	1	2	1	1	3	0	1	2	0
1	1	0	1	0	0	1	1	1	1	0	1
1	2	0	1	3	0	1	2	1	1	3	1
2	0	1	2	1	1	2	0	1	2	1	1
2	3	1	2	2	1	2	3	1	2	2	1
2	1	0	2	0	0	2	1	0	2	0	0
2	2	0	2	3	0	2	2	0	2	3	0

Replication 5 or *Design III*						*Replication* 6 or *Design VI*					
B_1			B_2			B_1			B_2		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	–	0	0	–	0	0	–	0	0	–	0
0	–	0	0	–	0	0	–	0	0	–	0
0	–	1	0	–	1	0	–	1	0	–	1
0	–	1	0	–	1	0	–	1	0	–	1
1	0	1	1	2	1	1	0	0	1	2	0
1	1	1	1	3	1	1	1	0	1	3	0
1	2	0	1	0	0	1	2	1	1	0	1
1	3	0	1	1	0	1	3	1	1	1	1
2	0	1	2	2	1	2	0	1	2	2	1
2	1	1	2	3	1	2	1	1	2	3	1
2	2	0	2	0	0	2	2	0	2	0	0
2	3	0	2	1	0	2	3	0	2	1	0